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A Broadband Digital Diode Phase Shifter Using a Matrix Interconnection of Quadrature Hybrid Couplers

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An analysis of 2-bit digital diode phase shifter using four PIN diodes and a matrix arrangement of four 3-dB quadrature hybrids has been performed. The 0°, 90°, 180°, and 270° differential phase states were achieved by utilizing the 90° phase relationship of the hybrid coupler outputs and proper combinations of open circuits and short circuits provided by the diodes with the appropriate bias. The phase state determined at which of two ports of the hybrid matrix the output occurred; thus an additional SPDT or transfer switch was needed to complete the phase

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SECURITY CLASSIFICATION OF THIS PAGE (*When Data Entered*) shifter. Broad bandwidths can be achieved, since the bandwidth of the phaser is limited by the hybrid bandwidth and the band over which the diode "open" and "short" can be made to track, plus the bandwidth of the single transfer switch. With the addition of a second transfer switch in conjunction with a 45° Schiffman constant phase shift section, a 3-bit, or 45° resolution, phase shifter can be obtained. The effects of certain component errors and imperfections on the performance of the phaser have been presented; a generalized method for an exact error analysis has been derived.

CONTENTS

A BROADBAND DIGITAL DIODE PHASE SHIFTER USING A MATRIX INTERCONNECTION OF QUADRATURE HYBRID COUPLERS

INTRODUCTION

There are numerous microwave applications that require 2- or 3-bit digital phase shifters operating over broad bandwidths. PIN diode devices are generally used for rapid switching between phase states. The three common types of digital diode phase shifters include the standard switched line type, the loaded line type, and the switched line type that uses Schiffman constant-phase-shift sections. Since the differential phase shift of the standard switched line phaser is proportional to frequency, its operational bandwidth is inherently limited if a flat phase shift is required. Loaded line phasers can achieve bandwidths greater than an octave, but only for low values of differential phase shift $\Delta\varphi$ [1]. For $\Delta\varphi > 90^\circ$, the loaded line phaser is limited to bandwidths of less than one octave. The switched line phaser that uses Schiffman sections is limited by the bandwidth of the Schiffman constant phase shift sections and the resonance in the "off" path [1,2].

When the 90° phase relationship between the two outputs of 3-dB quadrature hybrid couplers is used, it is possible to build a broadband 2-bit phaser that uses a minimum number of diodes but does not use Schiffman sections. The bandwidth is limited primarily by the bandwidth of the hybrid couplers which have been built to operate over several octaves, and the frequency response of PIN diodes that act as open circuits and short circuits.

CIRCUIT DESCRIPTION

A matrix-type interconnection of couplers is shown in Fig. 1, where each coupler is shown schematically as a forward wave coupler.

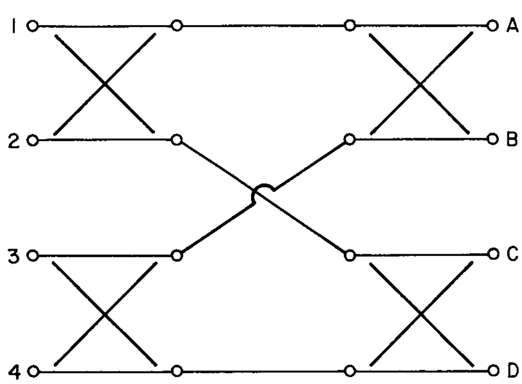

Fig. 1—Hybrid matrix interconnections

Note: Manuscript submitted December 12, 1974.

It may be assumed for the time being that each coupler is ideal; i.e., it is lossless, perfectly matched, has an even power split between the two outputs with exactly 90° phase difference, and infinite isolation at the isolated port when a single input is applied. The scattering matrix for each ideal 3-dB quadrature hybrid coupler is

$$[S] = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 0 & 1 & j \\ 0 & 0 & j & 1 \\ 1 & j & 0 & 0 \\ j & 1 & 0 & 0 \end{bmatrix},$$

where the four ports are labeled as in Fig. 2

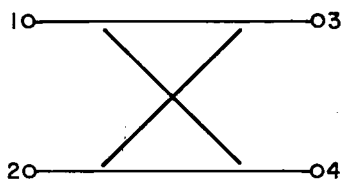

Fig. 2—Three decibel quadrature hybrid

Since the isolation and reflection component voltages are zero, we may consider only the voltage transfer terms. That is, for a single coupler

$$\begin{bmatrix} V_3 \\ V_4 \end{bmatrix} = [S_1] \begin{bmatrix} E_1 \\ E_2 \end{bmatrix} \quad \text{where} \quad [S_1] = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & j \\ j & 1 \end{bmatrix}.$$

Then for the matrix configuration of ideal couplers in Fig. 1, it is straightforward to show that

$$\begin{bmatrix} V_A \\ V_B \\ V_C \\ V_D \end{bmatrix} = \left[\begin{array}{c|c} S_1 & 0 \\ \hline 0 & S_1 \end{array}\right] \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \left[\begin{array}{c|c} S_1 & 0 \\ \hline 0 & S_1 \end{array}\right] \begin{bmatrix} E_1 \\ E_2 \\ E_3 \\ E_4 \end{bmatrix}$$

or

$$\begin{bmatrix} V_A \\ V_B \\ V_C \\ V_D \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & j & j & -1 \\ j & -1 & 1 & j \\ j & 1 & -1 & j \\ -1 & j & j & 1 \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \\ E_4 \end{bmatrix}.$$

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Define $[L]$ by

$$[L] \equiv \left[\begin{array}{c|c} S_1 & 0 \\ \hline 0 & S_1 \end{array}\right] \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \left[\begin{array}{c|c} S_1 & 0 \\ \hline 0 & S_1 \end{array}\right]$$

so that

$$[\mathbf{V}_{ABCD}] = \begin{bmatrix} V_A \\ V_B \\ V_C \\ V_D \end{bmatrix} = [L] \begin{bmatrix} E_1 \\ E_2 \\ E_3 \\ E_4 \end{bmatrix} = [L][\mathbf{E}_{1234}] \; .$$

Now $[L]^{-1} = [L]^*$. That is,

$$[\mathbf{E}_{1234}] = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \\ E_4 \end{bmatrix} = [L]^{-1} \begin{bmatrix} V_A \\ V_B \\ V_C \\ V_D \end{bmatrix} = [L]^{-1}[\mathbf{V}_{ABCD}]$$

where $[\mathbf{E}_{1234}]$ are the input voltages on the left side of Fig. 1 that cause $[\mathbf{V}_{ABCD}]$ on the right side. However, if voltages $[\mathbf{E}_{ABCD}]$ are applied on the right side, then by symmetry $[\mathbf{V}_{1234}] = [L][\mathbf{E}_{ABCD}]$ are the resultant output voltages on the left side.

If each output $\{A, B, C, D\}$ on the right side of Fig. 1 is terminated with a mismatch $\{\Gamma_A, \Gamma_B, \Gamma_C, \Gamma_D\}$, then at each port the input to that port will be Γ_X times the output from that port; i.e., $E_A = \Gamma_A, V_A$, etc. Thus, the resultant voltages out on the left side of Fig. 1 will be

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = [L] \begin{bmatrix} E_A \\ E_B \\ E_C \\ E_D \end{bmatrix} = [L] \begin{bmatrix} \Gamma_A & 0 & 0 & 0 \\ 0 & \Gamma_B & 0 & 0 \\ 0 & 0 & \Gamma_C & 0 \\ 0 & 0 & 0 & \Gamma_D \end{bmatrix} [L] \begin{bmatrix} E_1 \\ E_2 \\ E_3 \\ E_4 \end{bmatrix}$$

where E_i denotes the voltage *input* to the ith port and V_i denotes the voltage *output* from the ith port.

An input signal (normalized to 1 V) can be applied at only one input port, for example port 1. Elementary matrix multiplication yields

$$V_1 = \frac{1}{4}\,[\Gamma_A - \Gamma_B - \Gamma_C + \Gamma_D]$$

$$V_2 = \frac{1}{4}\,j[\Gamma_A - \Gamma_B + \Gamma_C - \Gamma_D]$$

$$V_3 = \frac{1}{4}\,j[\Gamma_A + \Gamma_B - \Gamma_C - \Gamma_D]$$

$$V_4 = \frac{1}{4}\,[-\Gamma_A - \Gamma_B - \Gamma_C - \Gamma_D]\,.$$

If Γ_X is restricted to ±1, then the outputs for the specified combinations of Γ_X would be as given in Table 1,

Table 1
Output Voltages for Input at Port 1

Γ_A	Γ_B	Γ_C	Γ_D	Voltage Out at Port 3	Voltage Out at Port 4
−1	−1	−1	−1	0	$1 \angle 0°$
+1	+1	+1	+1	0	$1 \angle 180°$
+1	+1	−1	−1	$1 \angle 90°$	0
−1	−1	+1	+1	$1 \angle 270°$	0

where the voltages out at ports 1 and 2 would be zero for all cases shown, and the 0° phase reference is arbitrary.

If the unit voltage input were applied at port 2 rather than at port 1, the output listing would be as in Table 2,

Table 2
Output Voltages for Input at Port 2

Γ_A	Γ_B	Γ_C	Γ_D	Voltage Out at Port 3	Voltage Out at Port 4
−1	−1	−1	−1	$1 \angle 0°$	0
+1	+1	+1	+1	$1 \angle 180°$	0
+1	+1	−1	−1	0	$1 \angle 270°$
−1	−1	+1	+1	0	$1 \angle 90°$

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where the reference (zero) phase angle is the same as that used for the case where the input was applied to port 1.

If the mismatch at each port $\{A, B, C, D\}$ is now implemented by terminating that port with a PIN diode, and if the diode is reversed biased, then the port will be open circuited to the RF and $\Gamma_X = +1$. If the diode is forward biased, the port will be short circuited to the RF and $\Gamma_X = -1$. This of course presupposes an ideal diode.

If a SPDT switch is then used to select port 3 or port 4 as the output, a 2-bit (0°, 90°, 180°, or 270°) digital phase shifter is obtained. In practice a transfer, or DPDT switch would be used to terminate the unused port in a matched load. The complete RF circuit for the phase shifter is shown in Fig. 3.

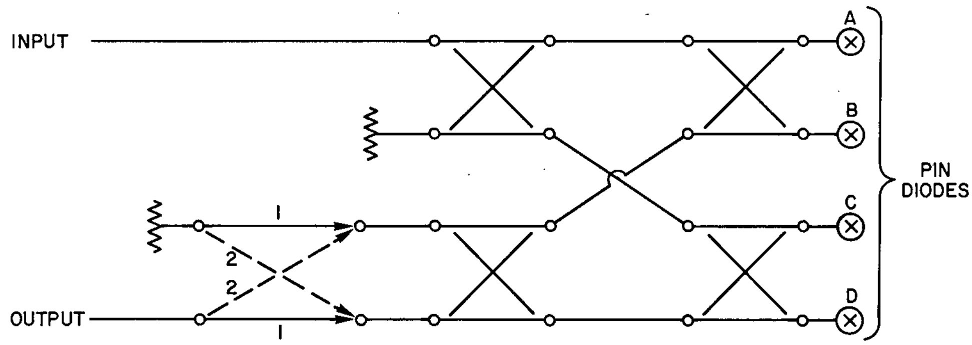

Fig. 3—Two-bit digital diode phase shifter

Bias circuits are required for the diodes and probably DC blocks would be required. The important feature of this phaser is that it is potentially very broadband; the bandwidth is limited by the hybrids, which can be built to cover several octaves, and the ability of the opposite diode states to track with frequency. The transfer switch is also a limitation, but these switches have been built to cover several octaves. There are no "off" paths to generate loss spikes at resonant frequencies. An additional advantage for the use of this phaser is that each of the four diodes terminating the coupler matrix handles only one-quarter of the total input power.

The diode biasing and transfer switch positions that are used to achieve various differential phase shifts for the phaser shown in Fig. 3 are listed in Table 3. This assumes

Table 3
Diode Biasing and Transfer Switch Settings for a 2-Bit Phase Shifter

Differential Phase Shift	Transfer Switch Position	Diode Biasing	
		A & *B*	*C* & *D*
0°	1	Forward	Forward
90°	2	Reverse	Forward
180°	1	Reverse	Reverse
270°	2	Forward	Reverse

no phase change between the two positions of the transfer switch. If the switch were of the phase-reversal type, i.e., 180° phase change between the two outputs for a given input, Table 3 would have to be modified by interchanging the 90° and 270° settings.

If an additional transfer switch and a 45° Schiffman constant phase shift section are used on the input as shown in Fig. 4, the phaser becomes 3-bit rather than 2-bit.

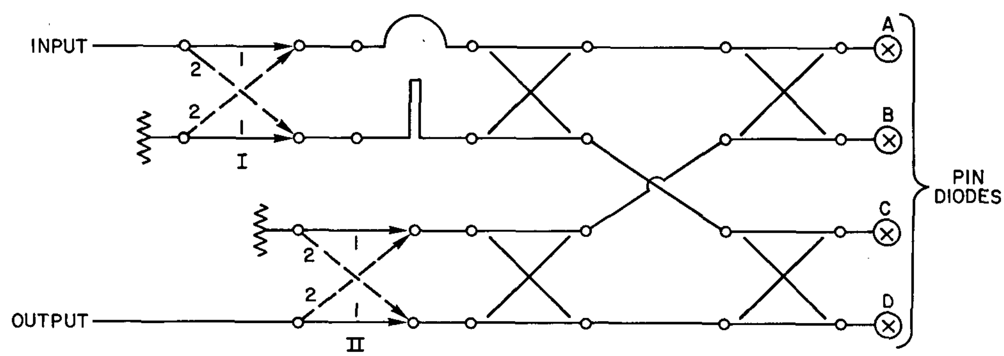

Fig. 4—Three-bit digital diode phase shifter

The diode biasing and transfer switch positions for different phase shifts are listed in Table 4. This listing again assumes transfer switches that do not reverse the phase; modification to have either or both switches of the phase reversal type is straightforward.

Table 4
Diode Biasing and Transfer Switch Settings for a 3-Bit Phase Shifter

Differential Phase Shift	Transfer Switch Position		Diode Biasing	
	I	II	*A & B*	*C & D*
0°	1	1	Forward	Forward
45°	2	2	Forward	Forward
90°	1	2	Reverse	Forward
135°	2	1	Forward	Reverse
180°	1	1	Reverse	Reverse
225°	2	2	Reverse	Reverse
270°	1	2	Forward	Reverse
315°	2	1	Reverse	Forward

The 0° reference state is arbitrary for both the 2-bit and the 3-bit phasers since the primary concern is the differential phase shift rather than the absolute phase. Also, both phasers are obviously reciprocal in operation.

GENERAL ERROR ANALYSIS

So far, the phase shifter has been built from ideal components. Since ideal components are somewhat difficult to obtain, the effects of real-world imperfections of the components upon the performance of the phaser must be considered. To do this, the scattering matrix must be calculated for the 2-port device obtained by terminating two ports of an imperfect 3-dB hybrid coupler with imperfect diodes (Fig. 5). The imperfect coupler can be characterized by a 4×4 scattering matrix $[S]$;

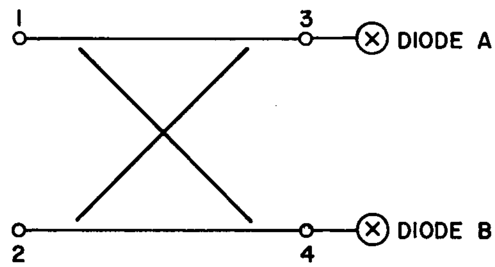

Fig. 5—Three-decibel hybrid with two diode terminations

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix} .$$

If the coupler is symmetrical, then

$$S_{11} = S_{22} = S_{33} = S_{44}$$
$$S_{12} = S_{21} = S_{34} = S_{43}$$
$$S_{13} = S_{31} = S_{24} = S_{42}$$
$$S_{14} = S_{41} = S_{23} = S_{32} ,$$

and the scattering matrix may be expressed in terms of the four quantities S_{11}, S_{12}, S_{13}, S_{14}. We shall work with the generalized scattering matrix; however, to simplify the matrix manipulation, let us partition $[S]$ into four 2×2 matrixes;

$$[S] = \left[\begin{array}{c|c} \theta_{11} & \theta_{12} \\ \hline \theta_{21} & \theta_{22} \end{array}\right] ,$$

where

$$[\theta_{11}] = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \qquad [\theta_{12}] = \begin{bmatrix} S_{13} & S_{14} \\ S_{23} & S_{24} \end{bmatrix}$$

$$[\theta_{21}] = \begin{bmatrix} S_{31} & S_{32} \\ S_{41} & S_{42} \end{bmatrix} \qquad [\theta_{22}] = \begin{bmatrix} S_{33} & S_{34} \\ S_{43} & S_{44} \end{bmatrix}.$$

Port 3 is terminated by diode A, which results in a reflection coefficient of Γ_A. Similarly, port 4 sees Γ_B. For real diodes $|\Gamma_X| < 1$.

Denote the input voltage at the ith port by E_i, and the output voltage by V_i. Then for the coupler

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = [S] \begin{bmatrix} E_1 \\ E_2 \\ E_3 \\ E_4 \end{bmatrix} = \left[\begin{array}{c|c} \theta_{11} & \theta_{12} \\ \hline \theta_{21} & \theta_{22} \end{array}\right] \left[\begin{array}{c} E_1 \\ E_2 \\ \hline E_3 \\ E_4 \end{array}\right].$$

But $E_3 = \Gamma_A V_3$ and $E_4 = \Gamma_B V_4$. Thus,

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \left[\begin{array}{c|c} \theta_{11} & \theta_{12} \\ \hline \theta_{21} & \theta_{22} \end{array}\right] \left[\begin{array}{c} E_1 \\ E_2 \\ \hline \Gamma_A V_3 \\ \Gamma_B V_4 \end{array}\right].$$

From this

$$\begin{bmatrix} V_3 \\ V_4 \end{bmatrix} = [\theta_{21}] \begin{bmatrix} E_1 \\ E_2 \end{bmatrix} + [\theta_{22}] \begin{bmatrix} \Gamma_A & 0 \\ 0 & \Gamma_B \end{bmatrix} \begin{bmatrix} V_3 \\ V_4 \end{bmatrix}.$$

Now define

$$[\Gamma_{AB}] \equiv \begin{bmatrix} \Gamma_A & 0 \\ 0 & \Gamma_B \end{bmatrix}.$$

Then

$$\begin{bmatrix} V_3 \\ V_4 \end{bmatrix} = [\theta_{21}] \begin{bmatrix} E_1 \\ E_2 \end{bmatrix} + [\theta_{22}][\Gamma_{AB}] \begin{bmatrix} V_3 \\ V_4 \end{bmatrix}$$

$$\Big[[I] - [\theta_{22}][\Gamma_{AB}] \Big] \begin{bmatrix} V_3 \\ V_4 \end{bmatrix} = [\theta_{21}] \begin{bmatrix} E_1 \\ E_2 \end{bmatrix}$$

$$\begin{bmatrix} V_3 \\ V_4 \end{bmatrix} = \Big[[I] - [\theta_{22}][\Gamma_{AB}] \Big]^{-1} [\theta_{21}] \begin{bmatrix} E_1 \\ E_2 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = [\theta_{11}] \begin{bmatrix} E_1 \\ E_2 \end{bmatrix} + [\theta_{12}] \begin{bmatrix} \Gamma_A & 0 \\ 0 & \Gamma_B \end{bmatrix} \begin{bmatrix} V_3 \\ V_4 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = [\theta_{11}] \begin{bmatrix} E_1 \\ E_2 \end{bmatrix} + [\theta_{12}][\Gamma_{AB}] \Big[[I] - [\theta_{22}][\Gamma_{AB}] \Big]^{-1} [\theta_{21}] \begin{bmatrix} E_1 \\ E_2 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = [M_{AB}] \begin{bmatrix} E_1 \\ E_2 \end{bmatrix}$$

where

$$[M_{AB}] = [\theta_{11}] + [\theta_{12}][\Gamma_{AB}] \Big[[I] - [\theta_{22}][\Gamma_{AB}] \Big]^{-1} [\theta_{21}]$$

is the 2×2 scattering matrix for the 2-port device shown in Fig. 5. The AB subscript indicates that the matrix is a function of the states of diodes A and B.

The circuit shown in Fig. 1 can be redrawn and each coupler labeled as shown in Fig. 6.

We wish to find the 4×4 scattering matrix for this 4-port circuit; i.e., we want $[R]$ such that

$$\begin{bmatrix} V_{\mathrm{I}} \\ V_{\mathrm{II}} \\ V_{\mathrm{III}} \\ V_{\mathrm{IV}} \end{bmatrix} = [R] \begin{bmatrix} E_{\mathrm{I}} \\ E_{\mathrm{II}} \\ E_{\mathrm{III}} \\ E_{\mathrm{IV}} \end{bmatrix}.$$

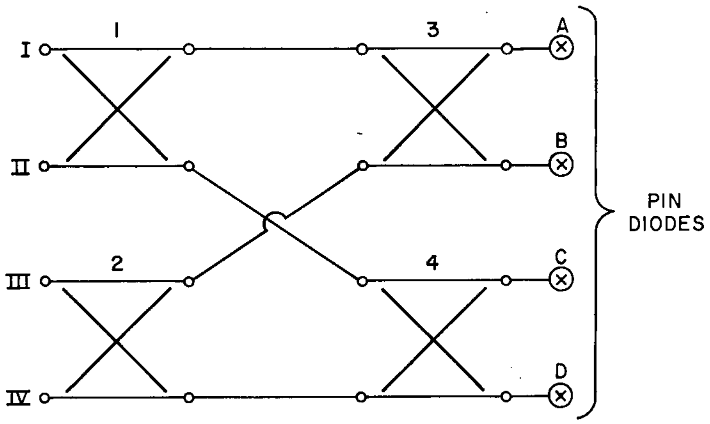

Fig. 6—Hybrid matrix with four diode terminations

Thus for any single input, the appropriate term in [R] will give the resultant voltage amplitude and phase at each output port. Since [R] is a function of the individual component parameters, including the diode reflection coefficients, the effect of less than ideal components on the phaser is found by calculating the reflected voltage at the input port to determine mismatch and the voltage at the output port for each desired value of phase shift. Which particular port serves as the output and the required diode biasing are determined by the desired phase shift. The different output voltage amplitudes and phases can be compared to determine loss, amplitude modulation, and differential phase shift error.

Denote E_i^j as the input voltage to the ith port of the jth coupler, where the coupler ports are numbered as in Fig. 2. Similarly, V_i^j is the output voltage from the ith port of the jth coupler. From the interconnections of Fig. 6, it is apparent

$$
\begin{aligned}
E_3^1 &= V_1^3 e^{-ja} & E_1^3 &= V_3^1 e^{-ja} \\
E_4^1 &= V_1^4 e^{-ja} & E_1^4 &= V_4^1 e^{-ja} \\
E_3^2 &= V_2^3 e^{-ja} & E_2^3 &= V_3^2 e^{-ja} \\
E_4^2 &= V_2^4 e^{-ja} & E_2^4 &= V_4^2 e^{-ja},
\end{aligned}
$$

where a is the electrical length of the interconnecting sections, which are assumed to be equal. If it is assumed that the four couplers are identical, they have the same scattering matrix [S];

$$
\begin{bmatrix} V_1^1 \\ V_2^1 \\ V_3^1 \\ V_4^1 \end{bmatrix} = [S] \begin{bmatrix} E_1^1 \\ E_2^1 \\ E_3^1 \\ E_4^1 \end{bmatrix} ; \qquad \begin{bmatrix} V_1^2 \\ V_2^2 \\ V_3^2 \\ V_4^2 \end{bmatrix} = [S] \begin{bmatrix} E_1^2 \\ E_2^2 \\ E_3^2 \\ E_4^2 \end{bmatrix} .
$$

These last two 4×4 matrix equations can now be combined into a single 8×8 matrix equation, as

$$\begin{bmatrix} V_1^1 \\ V_2^1 \\ V_3^1 \\ V_4^1 \\ V_1^2 \\ V_2^2 \\ V_3^2 \\ V_4^2 \end{bmatrix} = \boxed{\begin{array}{c|c} S & 0 \\ \hline 0 & S \end{array}} \begin{bmatrix} E_1^1 \\ E_2^1 \\ E_3^1 \\ E_4^1 \\ E_1^2 \\ E_2^2 \\ E_3^2 \\ E_4^2 \end{bmatrix} .$$

Define $[\Phi] \equiv \left[\begin{array}{c|c} S & 0 \\ \hline 0 & S \end{array}\right]$. Now use the relationship between E_i^j and V_1^k listed earlier;

$$\begin{bmatrix} V_1^1 \\ V_2^1 \\ V_3^1 \\ V_4^1 \\ V_1^2 \\ V_2^2 \\ V_3^2 \\ V_4^2 \end{bmatrix} = [\Phi] \begin{bmatrix} E_1^1 \\ E_2^1 \\ V_1^3 e^{-ja} \\ V_1^4 e^{-ja} \\ E_1^2 \\ E_2^2 \\ V_2^3 e^{-ja} \\ V_2^4 e^{-ja} \end{bmatrix}$$

A row transformation is now used.

$$\begin{bmatrix} E_1^1 \\ E_2^1 \\ E_1^2 \\ E_2^2 \\ V_1^3 e^{-ja} \\ V_2^3 e^{-ja} \\ V_1^4 e^{-ja} \\ V_2^4 e^{-ja} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} E_1^1 \\ E_2^1 \\ V_1^3 e^{-ja} \\ V_1^4 e^{-ja} \\ E_1^2 \\ E_2^2 \\ V_2^3 e^{-ja} \\ V_2^4 e^{-ja} \end{bmatrix} .$$

The 8×8 transform matrix is defined as $[Q]$. Since $[Q]^{-1} = [Q]^T$, we have

$$\begin{bmatrix} V_1^1 \\ V_2^1 \\ V_3^1 \\ V_4^1 \\ V_1^2 \\ V_2^2 \\ V_3^2 \\ V_4^2 \end{bmatrix} = [\Phi][Q]^T \boxed{\begin{array}{c|c} I & 0 \\ \hline 0 & a \end{array}} \begin{bmatrix} E_1^1 \\ E_2^1 \\ E_1^2 \\ E_2^2 \\ V_1^3 \\ V_2^3 \\ V_1^4 \\ V_2^4 \end{bmatrix}$$

where $[a] \equiv e^{-ja}[I]$. Using the results obtained for the 2×2 scattering matrix of a coupler terminated by two diodes, we see that

$$\begin{bmatrix} V_1^3 \\ V_2^3 \end{bmatrix} = [M_{AB}] \begin{bmatrix} E_1^3 \\ E_2^3 \end{bmatrix} = [M_{AB}] \begin{bmatrix} V_3^1 e^{-ja} \\ V_3^2 e^{-ja} \end{bmatrix}$$

$$\begin{bmatrix} V_1^4 \\ V_2^4 \end{bmatrix} = [M_{CD}] \begin{bmatrix} E_1^4 \\ E_2^4 \end{bmatrix} = [M_{CD}] \begin{bmatrix} V_4^1 e^{-ja} \\ V_4^2 e^{-ja} \end{bmatrix} .$$

Thus

$$\begin{bmatrix} E_1^1 \\ E_2^1 \\ E_1^2 \\ E_2^2 \\ V_1^3 \\ V_2^3 \\ V_1^4 \\ V_2^4 \end{bmatrix} \left[\begin{array}{c|cc} I & \multicolumn{2}{c}{0} \\ \hline 0 & M_{AB} & 0 \\ & 0 & M_{CD} \end{array}\right] \left[\begin{array}{c|c} I & 0 \\ \hline 0 & a \end{array}\right] \begin{bmatrix} E_1^1 \\ E_2^1 \\ E_1^2 \\ E_2^2 \\ V_3^1 \\ V_3^2 \\ V_4^1 \\ V_4^2 \end{bmatrix}$$

where as usual, $[I]$ is the unit matrix of the appropriate dimensions, in this case 4×4. We now have

$$\begin{bmatrix} V_1^1 \\ V_2^1 \\ V_3^1 \\ V_4^1 \\ V_1^2 \\ V_2^2 \\ V_3^2 \\ V_4^2 \end{bmatrix} = [\Phi][Q]^T \left[\begin{array}{c|c} I & 0 \\ \hline 0 & a \end{array}\right] \left[\begin{array}{c|cc} I & \multicolumn{2}{c}{0} \\ \hline 0 & M_{AB} & 0 \\ & 0 & M_{CD} \end{array}\right] \left[\begin{array}{c|c} I & 0 \\ \hline 0 & a \end{array}\right] \begin{bmatrix} E_1^1 \\ E_2^1 \\ E_1^2 \\ E_2^2 \\ V_3^1 \\ V_3^2 \\ V_4^1 \\ V_4^2 \end{bmatrix}.$$

Another row transformation matrix is used;

$$\begin{bmatrix} V_1^1 \\ V_2^1 \\ V_1^2 \\ V_2^2 \\ V_3^1 \\ V_3^2 \\ V_4^1 \\ V_4^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} V_1^1 \\ V_2^1 \\ V_3^1 \\ V_4^1 \\ V_1^2 \\ V_2^2 \\ V_3^2 \\ V_4^2 \end{bmatrix}.$$

This row transformation matrix is the same as the previous one, namely $[Q]$. Thus,

$$\begin{bmatrix} V_1^1 \\ V_2^1 \\ V_1^2 \\ V_2^2 \\ V_3^1 \\ V_3^2 \\ V_4^1 \\ V_4^2 \end{bmatrix} = [Q][\Phi][Q]^T \left[\begin{array}{c|c} I & 0 \\ \hline 0 & a \end{array}\right] \left[\begin{array}{c|cc} I & \multicolumn{2}{c}{0} \\ \hline 0 & \begin{array}{c|c} M_{AB} & 0 \\ \hline 0 & M_{CD} \end{array} \end{array}\right] \left[\begin{array}{c|c} I & 0 \\ \hline 0 & a \end{array}\right] \begin{bmatrix} E_1^1 \\ E_2^1 \\ E_1^2 \\ E_2^2 \\ V_3^1 \\ V_3^2 \\ V_4^1 \\ V_4^2 \end{bmatrix}$$

The following definitions are made to simplify the notation:

$$[P] \equiv [Q][\Phi][Q]^T \left[\begin{array}{c|c} I & 0 \\ \hline 0 & a \end{array}\right] \left[\begin{array}{c|c} I & 0 \\ \hline 0 & \begin{array}{c|c} M_{AB} & 0 \\ \hline 0 & M_{CD} \end{array} \end{array}\right] \left[\begin{array}{c|c} I & 0 \\ \hline 0 & a \end{array}\right]$$

$$[\mathbf{X}] \equiv \begin{bmatrix} V_1^1 \\ V_2^1 \\ V_1^2 \\ V_2^2 \end{bmatrix}, \quad [\mathbf{Y}] \equiv \begin{bmatrix} V_3^1 \\ V_3^2 \\ V_4^1 \\ V_4^2 \end{bmatrix}, \quad [\mathbf{E}] \equiv \begin{bmatrix} E_1^1 \\ E_2^1 \\ E_1^2 \\ E_2^2 \end{bmatrix}$$

Then

$$\begin{bmatrix} \mathbf{X} \\ \mathbf{Y} \end{bmatrix} = [P] \begin{bmatrix} \mathbf{E} \\ \mathbf{Y} \end{bmatrix}$$

To eliminate $[\mathbf{Y}]$ and to reduce to a 4×4 matrix equation, partition $[P]$ into four 4×4 submatrixes.

$$[P] = \left[\begin{array}{c|c} P_{11} & P_{12} \\ \hline P_{21} & P_{22} \end{array}\right]$$

$$[\mathsf{X}] = [P_{11}][\mathsf{E}] + [P_{12}][\mathsf{Y}]$$

$$[\mathsf{Y}] = [P_{21}][\mathsf{E}] + [P_{22}][\mathsf{Y}]$$

$$\Big[[I] - [P_{22}]\Big][\mathsf{Y}] = [P_{21}][\mathsf{E}]$$

$$[\mathsf{Y}] = \Big[[I] - [P_{22}]\Big]^{-1}[P_{21}][\mathsf{E}]$$

$$[\mathsf{X}] = [P_{11}][\mathsf{E}] + [P_{12}]\Big[[I] - [P_{22}]\Big]^{-1}[P_{21}][\mathsf{E}]$$

$$[\mathsf{X}] = [R][\mathsf{E}],$$

where

$$[R] = [P_{11}] + [P_{12}]\Big[[I] - [P_{22}]\Big]^{-1}[P_{21}] .$$

But

$$[\mathsf{X}] = \begin{bmatrix} V_1^1 \\ V_2^1 \\ V_1^2 \\ V_2^2 \end{bmatrix} = \begin{bmatrix} V_{\mathrm{I}} \\ V_{\mathrm{II}} \\ V_{\mathrm{III}} \\ V_{\mathrm{IV}} \end{bmatrix} \quad \text{and} \quad [\mathsf{E}] = \begin{bmatrix} E_1^1 \\ E_2^1 \\ E_1^2 \\ E_2^2 \end{bmatrix} = \begin{bmatrix} E_{\mathrm{I}} \\ E_{\mathrm{II}} \\ E_{\mathrm{III}} \\ E_{\mathrm{IV}} \end{bmatrix}$$

from Fig. 3. Thus $[R]$ is the scattering matrix initially desired. If an input signal is applied to only one port, such as port 1, then using $[R]$ we find the output at each port in terms of $[R]$, thusly in terms of the diode states $\{\Gamma_A, \Gamma_B, \Gamma_C, \Gamma_D\}$, and the scattering matrix for the couplers $[S]$. It has been assumed that the couplers are identical; however, the extension of the above development to include the case where the couplers have different scattering matrixes is straightforward. Also, it has been assumed that interconnecting lines between couplers are equal in length. If they were not, it would be necessary to modify the earlier voltage relationships between the input and output voltages to

$$\begin{aligned} E_3^1 &= V_1^3 e^{-ja_1} & E_1^3 &= V_3^1 e^{-ja_1} \\ E_4^1 &= V_1^4 e^{-ja_2} & E_1^4 &= V_4^1 e^{-ja_2} \\ E_3^2 &= V_2^3 e^{-ja_3} & E_2^3 &= V_3^2 e^{-ja_3} \\ E_4^2 &= V_2^4 e^{-ja_4} & E_2^4 &= V_4^2 e^{-ja_4} , \end{aligned}$$

where a_i denotes the electrical lengths of the interconnections. Again, the modifications to the earlier result would be straightforward.

The desired result $[R]$ has been obtained in terms of matrixes which have been defined from other more complicated matrixes, which have, in turn, been defined by even more complicated matrixes. To obtain $[R]$ directly in terms of the various components of the individual coupler scattering matrixes and the diode reflection coefficients would be a rather arduous task. However, numerical solutions can readily be obtained by the use of a computer.

SPECIFIC ERROR ANALYSIS

We can find $[R]$ directly if we assume that the couplers are identical and ideal except for an unbalance in the outputs. The scattering matrix of each coupler would be

$$[S] = \begin{bmatrix} 0 & 0 & T & jC \\ 0 & 0 & jC & T \\ T & jC & 0 & 0 \\ jC & T & 0 & 0 \end{bmatrix} .$$

Using $[\theta_{11}] = [\theta_{22}] = [0]$ and $[\theta_{12}] = [\theta_{21}] = \begin{bmatrix} T & jC \\ jC & T \end{bmatrix}$, we can insert these values in our earlier derivations to obtain $[R]$. If a unit voltage is applied to only one input, such as input 1, then $V_i = R_{i1}E_1 = R_{i1}$ is obtained, where $i = 1, 2, 3, 4$.

Identical results are obtained more simply by tracing through Fig. 6 and keeping track of the voltage (phase and amplitude) as the four possible paths from the input to the output in question are traced. Using this technique, one quickly obtains

$$\begin{aligned} V_1 &= T^4\Gamma_A - T^2C^2\Gamma_B - T^2C^2\Gamma_C + C^4\Gamma_D \\ V_2 &= j\left[T^3C\Gamma_A - TC^3\Gamma_B + T^3C\Gamma_C - TC^3\Gamma_D\right] \\ V_3 &= j\left[T^3C\Gamma_A + T^3C\Gamma_B - TC^3\Gamma_C - TC^3\Gamma_D\right] \\ V_4 &= -\left[T^2C^2\Gamma_A + T^2C^2\Gamma_B + T^2C^2\Gamma_C + T^2C^2\Gamma_D\right] . \end{aligned}$$

If the couplers are ideal, then $T^2 + C^2 = 1$ and $T^2 = C^2 = 1/2$ for perfect balance in the outputs. Denote by $(P_i)j$ the normalized power at the ith port when the diode states are chosen from Table 1 to give the output (in the idealized case) at the jth port. Port 2 is unused and terminates in a matched load, and will not be considered. The two desired outputs are ports 3 and 4; when port 3 is selected (for either of its two phase settings) port 4 will be terminated, and vice-versa. The reflected power at the input is $(P_1)j$, where $j = 3, 4$. If ideal diodes are assumed, $\Gamma_x = \pm 1$; thus

$$(P_1)_3 = (T^2 - C^2)^2$$

$$(P_1)_4 = (T^2 - C^2)^4$$

$$(P_3)_3 = 4T^2C^2$$

$$(P_4)_4 = (4T^2C^2)^2 .$$

Since $|T^2 - C^2| < 1$, the worst-case reflection occurs when port 3 is chosen as the output. Also, the transmission loss to port 4 is twice the transmission loss to port 3, expressed in decibels, since $(P_4)_4 = [(P_3)_3]^2$. Therefore, $(P_3)_3$ represents the possible amplitude modulation of the signal as diode states are switched to change the phase. These quantities are expressed in decibels and plotted as a function of power coupling C^2 in Fig. 7.

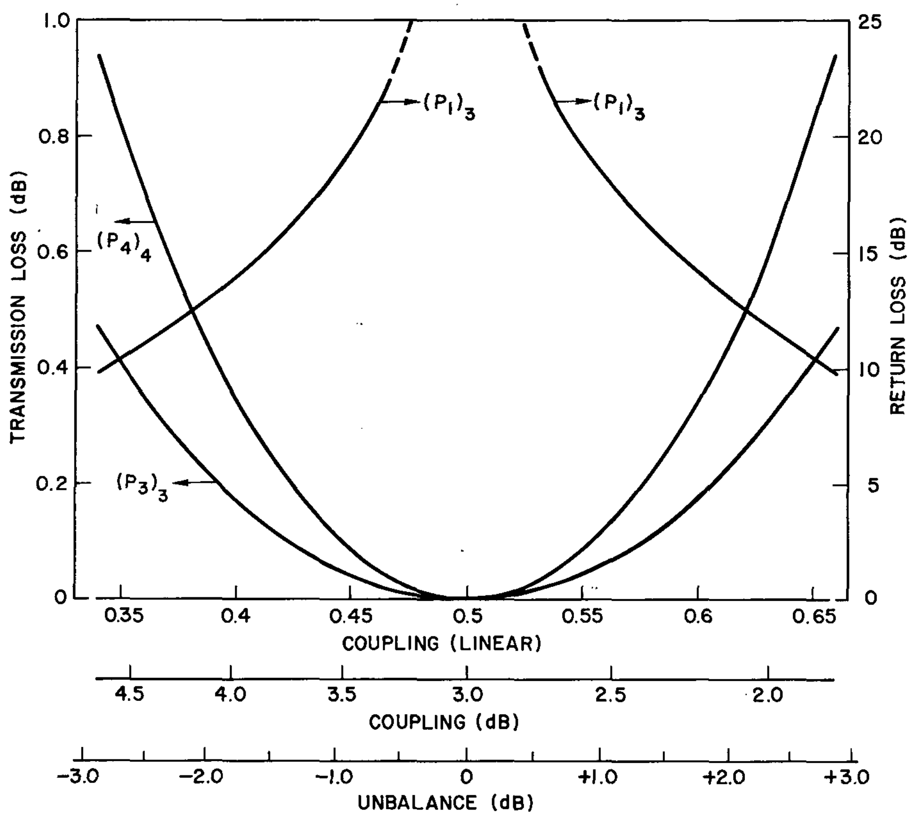

Fig. 7—Effect of hybrid unbalance on phaser loss

If other components are ideal, the hybrid unbalance does not cause phase error. It should be emphasized that the transmission and return losses shown in Fig. 7 are the consequence of hybrid unbalance only.

Loss and amplitude modulation are also caused by departure from the ideal open/short conditions of any real diodes. For a forward-biased diode, denote the reflection coefficient $\Gamma_f = A_f \exp(ja_f)$; for a reverse-biased diode denote $\Gamma_r = A_r \exp(ja_r)$. Ideally, $A_f = A_r = 1$, $a_f = \pi$, and $a_r = 0$. In practice, $A_f < 1$, $A_r < 1$, $a_f = \pi + \epsilon_f$, and $a_r = 0 + \epsilon_r$. Since the diode states occur in pairs, i.e., both diodes terminating the output ports of either quadrature hybrid have the same bias, then for ideal hybrid couplers with equal power split,

$$V_4(\Delta\varphi_{\mathrm{I}} = 0^\circ) = -A_f e^{ja_f}$$

$$V_4(\Delta\varphi_{\mathrm{I}} = 180^\circ) = -A_r e^{ja_r}$$

$$V_3(\Delta\varphi_{\mathrm{I}} = 90^\circ) = -j\,\frac{1}{2}\left[A_f e^{ja_f} - A_r e^{ja_r}\right]$$

$$V_3(\Delta\varphi_{\mathrm{I}} = 270^\circ) = j\,\frac{1}{2}\left[A_f e^{ja_f} - A_r e^{ja_r}\right]$$

where $V_i(\Delta\varphi_{\mathrm{I}} = \psi)$ denotes the voltage out at the ith port when the diode states are chosen from Table 1 to give a differential phase shift ψ under ideal conditions. These relationships for the output voltages assume that all diodes in the same state, i.e., having the same bias, have identical reflection coefficients.

It is easy to see that if the phase errors of the diode reflection coefficients are zero, then magnitudes less than unity will cause no phase errors in the phasor. The maximum transmission loss due to the diode loss will be

$$\text{Maximum Loss (dB)} = -20\log_{10}(\text{smaller of } \{A_r, A_f\}).$$

Loss is shown as a function of reflection coefficient magnitude in Fig. 8. The minimum transmission loss will be

$$\text{Minimum Loss (dB)} = -20\log_{10}(\text{larger of } \{A_r, A_f\}).$$

The 90° and 270° phase states will have a transmission loss between these two values, since if $A < B$, then

$$A^2 < \frac{1}{4}(A+B)^2 < B^2 .$$

The amplitude modulation of the phaser will be the difference between the maximum and minimum values of transmission loss.

Phase errors, i.e., departures from the phases of an ideal open and short, of the diode reflection coefficients cause both amplitude loss and phase errors in the output of the phaser. If unity magnitude for the reflection coefficients and ideal quadrature couplers with zero unbalance are assumed, then

$$V_4(\Delta\psi_{\mathrm{I}} = 0^\circ) = -e^{ja_f} = 1 \angle\, \epsilon_f$$

$$V_4(\Delta\psi_{\mathrm{I}} = 180^\circ) = -e^{ja_r} = 1 \angle\, 180^\circ + \epsilon_r$$

$$V_3(\Delta\psi_{\mathrm{I}} = 90^\circ) = -j\,\frac{1}{2}\left(e^{ja_f} - e^{ja_r}\right)$$

$$= \cos\left(\frac{\epsilon_f - \epsilon_r}{2}\right) \angle\, 90^\circ + \frac{\epsilon_f + \epsilon_r}{2}$$

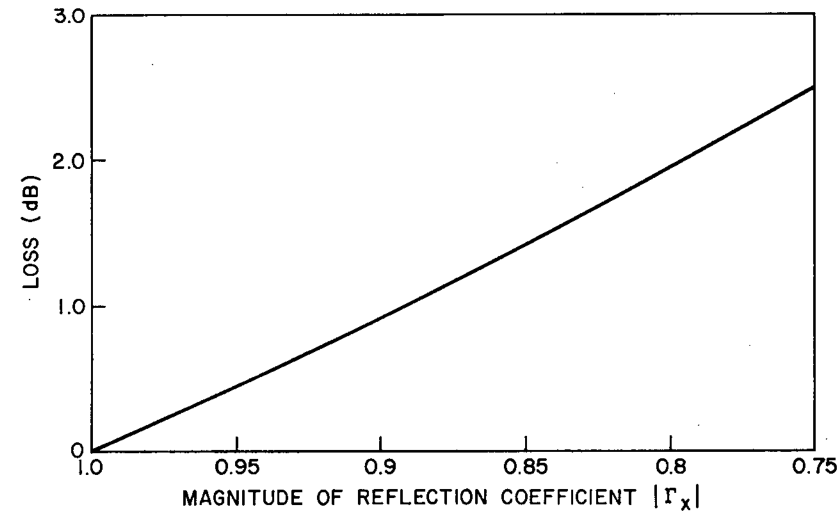

Fig. 8—Phaser transmission loss vs diode reflection coefficient magnitude

$$V_3(\Delta\psi_I = 270^\circ) = +j\,\frac{1}{2}\left(e^{ja_f} - e^{ja_r}\right)$$

$$= \cos\left(\frac{\epsilon_f - \epsilon_r}{2}\right) \angle\ 270^\circ + \frac{\epsilon_f + \epsilon_r}{2}\,.$$

Thus

$$\text{Maximum Loss (dB)} = -20\log_{10}\left[\cos\left(\frac{\epsilon_f - \epsilon_r}{2}\right)\right]$$

and minimum loss (dB) = 0. Note that if $\epsilon_r = \epsilon_f = \epsilon$, then the transmission loss is zero and the differential phase shift error is also zero since each phase state is shifted by the same amount.

It is apparent that if the forward- and reverse-bias reflection coefficients of the diode were plotted on a Smith chart, and the resulting "short" and "open" had equal magnitudes (less than unity) but were 180° apart, corresponding to $\epsilon_r = \epsilon_f$, the phaser would be lossy but would have no amplitude modulation and no differential phase error. Since a practical diode will typically be inductive when forward biased and capacitive when reverse biased, the respective phase errors ϵ_f and ϵ_r will have the same sign (negative). A diode with the correct parasitics will give $\Gamma_f = -\Gamma_r$; therefore, $A_f = A_r$ and $\epsilon_f = \epsilon_r$. This is at a single frequency, of course. The techniques for making the "open" and "short" track, i.e., $\Gamma_f = -\Gamma_r$, over a broad frequency band are beyond the scope of this report; however, the same tracking techniques [3] used for 180° diode phase shifters are applicable.

Other component imperfections that will affect the phaser performance include the isolation, mismatch, transmission loss, and departure from phase quadrature of the hybrids, variation of interconnecting line lengths, as well as transfer switch characteristics. The latter has not been included in the error analysis since the transfer switch could be any one of several types; its performance limitations would be added to those of the matrix/diode array since it is in series. The effect of hybrid loss is obvious; the effects of other component errors or limitations are generally interrelated, and a solution would best be obtained by the use of a computer implementing the general error analysis described earlier with specific component parameters.

CONCLUSION

The phase shifter described in this report is similar to the conventional 180° phase shifter obtained by terminating two ports of a 3-dB coupler with PIN diodes and biasing the diodes to obtain effective open or short circuits. By using 4-phase quadrature, 3-dB couplers in a matrix arrangement to form an 8-port device and by terminating four ports with appropriate diode opens and shorts, 2-bit or 90° phase resolution is obtained. A transfer switch must be used to complete the phaser since the output port depends upon the phase state. By incorporating one additional transfer switch and a 45° Schiffman constant phase shift section, 3-bit or 45° resolution is obtained.

The interesting feature of this phase shifter is that it achieves the 90° resolution by using the phase quadrature output relationship of the 3-dB hybrid couplers. With reasonably well-matched hybrids there are no high-Q resonant cavities to cause loss spikes. The operational bandwidth is potentially very large, the principal limitations being hybrid bandwidth and frequency sensitivity of the diode open and short states.

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